A Workshop on Applied and Numerical Mathematics

on the Occasion of Professor Bengt Fornberg's 80th Birthday

Monday June 8th through Wednesday June 10th 2026

Michigan Technological University

Sponsored by the Igor Kliakhandler Fellowship
and by the Applied Mathematics Department at CU Boulder

Participants

Bengt Fornberg
Professor Emeritus, University of Colorado Boulder

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Professor of Applied Mathematics, Harvard

Mark Ablowitz
Distinguished Professor, University of Colorado Boulder

André Weideman
Distinguished Professor, Stellenbosch University, South Africa

Natasha Flyer
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Greg Fasshauer
Professor, Applied Mathematics and Statistics, Colorado School of Mines

Elisabeth Larsson
Professor in the Department of Information Technology, Uppsala University, Sweden

Toby Driscoll
Professor in the Department of Mathematical Sciences, University of Delaware

Ziad Musslimani
Professor in the Department of Mathematics, Florida State University

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Grady Wright
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Associate Professor, Department of Physics, University of Crete, Greece

Cecile Piret
Workshop Organizer, Associate Professor, Michigan Technological University

	Monday June 8, 2026	**Tuesday** June 9, 2026	**Wednesday** June 10, 2026
9:00 - 9:30	The continuing contributions to numerical quadrature by James Gregory (1638-1675) (p.7) Bengt Fornberg	Pade Approximation Methods and Ordinary Differential Equations (p.14) Jonah Reeger	Radial basis function-generated finite differences for solving elliptic PDEs (p.12) Morten Eggert Nielsen
9:30 - 10:00	Computing with rational functions (p.16) Nick Trefethen	Quadrature formulas arising from discretizations of boundary value problems (p.18) Grady Wright	Fractional Derivatives - Definitions, Approximation, and Applications (p.9) Andrew Lawrence
10:00 - 10:30	**Coffee Break**	**Coffee Break**	**Coffee Break**
10:30 - 11:00	The Gibbs phenomenon revisited (p.17) André Weideman	Flexible patch-based high-order methods for solving PDEs (p.8) Elisabeth Larsson	Continued fractions for rational interpolation (p.5) Toby Driscoll
11:00 - 11:30	A Big Picture of Kernels: Looking at Three Views of Kernels - Starting from about 100 Years of Foundation Work (p.6) Greg Fasshauer	Rectangular spectral collocation (p.5) Toby Driscoll	Computing Differential Operators on Manifolds using The Radial Basis Functions Method (p.13) Cécile Piret
11:30 - 1:30	**Lunch Break**	**Lunch Break**	**Lunch Break**
1:30 - 2:00	A new numerical method for the Gauss hypergeometric function (p.7) Bengt Fornberg	The Pole Field Solver (p.17) André Weideman	3D harmonic-function specific FD methods for differentiation of aeromagnetic data (p.15) Jeff Thurston
2:00 - 2:30	Spiral waves in dispersive wave systems (p.4) Mark Ablowitz	A new framework for quadrature in geometrically complex domains (p.18) Grady Wright	A Big Picture of Kernels: Looking at Three Views of Kernels - Learning, Randomization and Explainability (p.6) Greg Fasshauer
2:30 - 3:00	**Coffee Break**	**Coffee Break**	**Coffee Break**
3:00 - 3:30	Quadrature = rational approximation (p.16) Nick Trefethen	Non-normality, pseudospectra, and singular values in non-Hermitian Photonics (p.10) Konstantinos Makris	Recent Developments in Approximation of Definite Integrals with Radial Basis Functions (p.14) Jonah Reeger
3:30 - 4:00	On optimal shape parameters in Gaussian radial basis function approximation (p.8) Elisabeth Larsson	Time dependent Duhamel renormalization method (p.11) Ziad Musslimani	High-Order Numerical Solver for Fractional Order Initial Value Problems (p.9) Andrew Lawrence
6:00 - 8:00	Conference Dinner	Dinner	Dinner

Conference Schedule. Educational lectures are highlighted in blue, while research presentations are shown in green.

Abstracts

Mark Ablowitz
Distinguished Professor, University of Colorado Boulder

Research Talk: Spiral waves in dispersive wave systems
Monday 2:00-2:30

After a brief discussion of recent research, focus of the talk will be on spiral waves. While spiral waves in excitable nonlinear media are well-known, they have not been carefully studied in fundamental linear/nonlinear dispersive wave systems. Recent results re: spiral waves in linear and nonlinear Dirac systems, Klein-Gordon equations and classical irrotational water waves will be discussed. Spiral generation in water waves is remarkably similar to the well-known problem of dropping a pebble in a still pond. If time permits density wave models of spiral galaxies will also be discussed.

Toby Driscoll
Professor in the Department of Mathematical Sciences, University of Delaware

Lecture 1: Rectangular spectral collocation
Tuesday 11:00-11:30

Introductions to spectral collocation (pseudospectral) methods suggest imposing boundary conditions by replacing rows in the differential operator. This idea is difficult to generalize past scalar. second-order problems. Treating spectral differentiation as a rectangular, not square, process leads to a more natural and robust alternative.

Lecture 2: Continued fractions for rational interpolation
Wednesday 10:30-11:00

A 1909 treatise by Thiele on the use of continued fractions to interpolate function values sat largely ignored for numerical computation due to instability. But in 2024, Oliver Salazar Celis noted that greedy node selection like in the AAA algorithm appears to allow stable use of Thiele's formula. Subsequent improvements and implementation have demonstrated that it is faster both asymptotically and in practice than AAA, while requiring only simple arithmetic to implement.

Greg Fasshauer

Professor, Applied Mathematics and Statistics, Colorado School of Mines

Lecture: A Big Picture of Kernels: Looking at Three Views of Kernels - Starting from about 100 Years of Foundation Work
Monday 11:00-11:30

This "big picture" will start by looking at three general views of kernels:

- Reproducing Kernel Hilbert Spaces (RKHSs), i.e., functional analysis
- Feature Maps, i.e., statistical learning
- Gaussian Processes (GPs), i.e., probabilistic.

Next, we will list both Mercer and Bochner theorems and look at general ridge regression problems.

Research Talk: A Big Picture of Kernels: Looking at Three Views of Kernels - Learning, Randomization and Explainability
Wednesday 2:00-2:30

Using both deterministic and probability versions we can talk about the most recent/modern ideas, namely random Fourier features and Large Language Models (LLMs). This more general thinks about learning, randomization and explainability.

Bengt Fornberg

Professor Emeritus, University of Colorado Boulder

Lecture: The continuing contributions to numerical quadrature by James Gregory (1638-1675)

Monday 9:00-9:30

Over 350 years have passed since James Gregory's early death at age 36. He is still well remembered in the astronomical community as a pioneering telescope designer but is mostly forgotten among mathematicians as a pioneer of calculus (where he made his contributions over a decade before the topic was described by Leibniz and Newton). We will in this presentation first note some of Gregory's works in both astronomy and mathematics, before concentrating on his method for numerical quadrature. This method is in most ways superior to the (later) Newton-Cotes approach that nowadays typically is focused on in numerical textbooks. In the last few years, Gregory's approach has led to several additional developments, including very high-accuracy methods for (i) quadrature on equispaced grids (with all weights positive), (ii) integration of discontinuous functions, (iii) contour integration in the complex plane, and (iv) numerical evaluation of fractional derivatives.

Research Talk: A new numerical method for the Gauss hypergeometric function

Monday 1:30-2:00

Among the multitude of special functions, the Gauss 2F1 hypergeometric function stands out not only in terms of how many other commonly used functions it contains as special cases, but also in terms of the wide range of important applications where it cannot readily be expressed by different means. Available numerical methods for evaluating it across the complex plane often depend heavily on the quite large number of functional identities that it satisfies, but these methods are nevertheless often quite inaccurate. This issue has in some cases been addressed by resorting to extended precision arithmetic, significantly increasing the computational cost. Key to the method that we will describe here is an elementary conformal mapping that surprisingly has not been utilized previously in this context. While our focus is on the 2F1 function, this mapping is applicable also to some other families of special functions. The present work was carried out in collaboration with Caleb Jacobs and Cécile Piret.

Elisabeth Larsson
Professor in the Department of Information Technology, Uppsala University, Sweden

Lecture : Flexible patch-based high-order methods for solving PDEs
Tuesday 10:30-11:00

Partition of unity methods divide a large computational problem into a set of coupled local problems defined on overlapping patches. By allowing the local problems to be unfitted and using least-squares collocation, which can also be seen as an inexact quadrature method, we construct a rather flexible method that can easily be applied to PDEs in complex geometries. The focus in this presentation will be on such methods using local radial basis function approximations. These are suitable when the PDE problem has a smooth solution and allow for global high-order convergence.

Research Talk: On optimal shape parameters in Gaussian radial basis function approximation
Monday 3:30-4:00

This talk will provide some theoretical insights into why sometimes a specific shape of Gaussian radial basis functions (RBFs) provides a significantly more accurate approximation than other nearby shapes. The discussion will be based on a recently derived expression for the Gaussian RBF interpolant in the small shape parameter range, which exposes the role of the shape parameter in the approximation error as well as the relation between RBF interpolation and polynomial interpolation. Based on these insights, also computational approaches to determining the optimal shape parameter will be discussed. Examples at least for one-dimensional cases will be presented.

Andrew Lawrence
Assistant Professor of Mathematics, Air Force Institute of Technology

Lecture : Fractional Derivatives - Definitions, Approximation, and Applications
Wednesday 9:30-10:00

Introduction to fractional calculus discussing the Gamma function, Riemann-Liouville integration, and Riemann-Liouville, Grünwald-Letnikov, and Caputo derivatives. Applications to fractional differential equations are presented with real-world applications. Solution methods including Laplace Transforms and approximation methods are presented.

Research Talk: High-Order Numerical Solver for Fractional Order Initial Value Problems
Wednesday 3:30-4:00

A novel method for the numerical solution to fractional initial value problems is presented. This method is stable for an accuracy of up to 6th order. The approach combines backward differentiation time stepping with a generalization of the classical Gregory method for numerical quadrature. Techniques for initializing the domain without loss of accuracy and suppressing unknown base point singularities are included. Computational results are given for a wide array of test problems.

Konstantinos Makris

Associate Professor, Department of Physics, University of Crete, Greece

Research Talk: Non-normality, pseudospectra, and singular values in non-Hermitian Photonics
Tuesday 3:00-3:30

One of the areas of physics where non-normality is evident is that of optics. In this context of non-Hermitian photonics, which the last 20 years is one of the frontiers of optical physics, we systematically describe extreme power dynamics and spectral ultra-sensitivity in coupled waveguides/cavities that either contain gain- loss or asymmetric couplings, based on the unified framework of pseudospectra-singular values. Recent results regarding higher order exceptional points, exponential sensitivity and transient power growth in optics, will be discussed. A general view of the field and relevant experimental results will also be presented.

Ziad Musslimani
Professor in the Department of Mathematics, Florida State University

Research Talk: Time dependent Duhamel renormalization method
Tuesday 3:30-4:00

In this talk, we introduce a time-dependent Duhamel renormalization method as a numerical means to simulate linear and nonlinear evolution equations. The essence of the method is to convert the underlying evolution equation from its partial or ordinary differential form (using Duhamel's principle) into an integral equation. The solution sought is then viewed as a fixed point in both space and time. The resulting integral equation is then numerically solved using a simple renormalized fixed-point iteration method. Convergence is achieved by introducing a time-dependent renormalization factor which is numerically computed from the physical properties of the governing evolution equation. The proposed method has the ability to incorporate physics into the simulations in the form of conservation law.

Morten Eggert Nielsen

Scientific software specialist at Danish Hydraulic Institute (DHS A/S), Hørsholm, Denmark

Lecture: Radial basis function-generated finite differences for solving elliptic PDEs
Wednesday 9:00-9:30

This talk will introduce radial basis function-generated finite differences with emphasis on solving elliptic partial differential equations. Initially, the talk is motivated by water wave modelling. Hereafter, the focus will be on water wave propagation formulated as either linear or nonlinear potential flow problems, which requires repeated solutions of the Laplace equation in either a time-invariant or time-varying domain. A meshfree methodology will be presented for discretizing and solving potential flow problems using radial basis function-generated finite differences. Furthermore, the linear problem will be used to illustrate how linear stability analysis can be used in relation to temporal stability, while the nonlinear problem will be used to illustrate the possibilities of dealing with time-varying domains.

Cécile Piret

Workshop Organizer, Associate Professor, Michigan Technological University

Lecture : Computing Differential Operators on Manifolds using The Radial Basis Functions Method
Wednesday 11:00-11:30

In this lecture, I will present the tensor-based approach to the discretization of manifold-constrained differential operators. The lecture will begin with an introduction to tensors and their role in differential geometry, followed by the derivation of fundamental formulas for differential operators on manifolds. I will then discuss how the geometric flexibility of radial basis functions provides a natural framework for discretizing these operators on general surfaces and manifolds. The lecture will conclude with a selection of applications illustrating the power and versatility of these techniques.

Jonah Reeger

Associate Professor of Mathematics, Air Force Institute of Technology

Lecture : Pade Approximation Methods and Ordinary Differential Equations
Tuesday 9:00-9:30

Taylor polynomial based methods for solving ODEs break down when the radius of convergence of the Taylor series is violated when taking a step in the solution of an initial value problem. This talk explores an alternative method that relies on the Pade approximation and a choice of complex step direction to avoid violating the radius of convergence when it is limited by the location of a pole.

Research Talk: Recent Developments in Approximation of Definite Integrals with Radial Basis Functions
Wednesday 3:00-3:30

Quadrature routines in one-dimension and on prescribed sets of nodes (e.g., uniformly spaced or roots of orthogonal polynomials) are well established. When moving beyond these cases to higher dimensions or to scattered data, a user is often required to perform an intermediate interpolation step to align the nodes in each dimension to those of an established 1-D quadrature rule, with the definite integral then approximated by a tensor product formula. This talk explores recent advances, including adaptive node refinement to achieve a prescribed estimated error, in approximating definite integrals over bounded areas, surfaces and volumes that avoid this intermediate interpolation step. Someof this is joint work that began with Bengt in 2013.

Jeff Thurston
Consultant, Geophysical Prospecting, Calgary, Alberta, Canada

Research Talk: 3D harmonic-function specific FD methods for differentiation of aeromagnetic data
Wednesday 1:30-2:00

We adapt a new class of harmonic-specific finite difference (FD) stencils to numerical differentiation of aeromagnetic grids using 3D stencils. These comprise three vertical levels and measure 5x5 in the horizontal plane (denoted as 5x5x3 operators). They can be used for high accuracy estimates of first and second derivatives with respect to all three coordinate axes (five independent derivatives in all). The method is straightforward to implement using pre-existing functionality built into most potential-field software packages.
These 3D stencils require data above and below the measurement surface. Non-local (e.g. odd-order vertical derivative operators outperform classical methods when actual, as opposed to numerically estimated, off-plane data are used. Motivated by this, we explore applications to scalar vertical gradiometry.

In interpretation contexts local quantities estimated by the 3D FD stencils are shown to offer superior resolution compared to popular low accuracy alternatives We demonstrate this with two examples using horizontal gradients estimated from modern aeromagnetic data over north-central Alberta. The first study shows instances of seismically resolved faulted strata that are geometrically related to prominent features on gradients estimated using the 3D stencils. We also use the novel methods to interpret shallow paleochannels. In both cases these features remain invisible on horizontal gradients estimated by conventional FD methods.

Nick Trefethen
Professor of Applied Mathematics, Harvard

Lecture : Computing with rational functions
Monday 9:30-10:00

It's amazing what you can do computationally with rational functions (using the AAA algorithm). This talk will explore about a dozen examples.

Research Talk: Quadrature = rational approximation
Monday 3:00-3:30

Whenever you see a string of quadrature nodes, you can consider it as a branch cut defined by the poles of a rational approximation to the Cauchy transform of a weight function. The aim of this talk is to explain this statement and show how it opens the way to calculation of targeted quadrature formulas for all kinds of applications. Gauss quadrature is an example, but it is just the starting point, and many more examples will be shown. I hope this talk will change your understanding of quadrature formulas. This is joint work with Andrew Horning.

Andre Weideman
Distinguished Professor, Stellenbosch University, South Africa

Lecture : The Gibbs phenomenon revisited
Monday 10:30-11:00

The Gibbs phenomenon--the oscillations that arise when a function with a discontinuity is approximated by most series expansions--has been of interest to mathematicians, physicists, and engineers alike. Major efforts have gone into attempts at suppressing the oscillations. Some of these strategies will be reviewed in this talk, with special emphasis on the Fourier-Pade approximation method applied to the sawtooth function.

Research Talk: The Pole Field Solver
Tuesday 1:30-2:00

The Painleve family consists of six nonlinear ordinary differential equations. Their solutions are often characterized by dense pole fields in the complex plane, posing a significant challenge to numerical methods. A solver that could deal efficiently with this was developed by Bengt Fornberg and the speaker in the early 2010s. In this talk we review the ingredients of the solver, as well as some of its successes.

Grady Wright

Professor in the Department of Mathematics, Boise State University

Lecture : Quadrature formulas arising from discretizations of boundary value problems
Monday 9:30-10:00

The dominant approach to constructing quadrature formulas is to select a “nice” function space on which the formulas are exact, such as algebraic or trigonometric polynomials. In one dimension, this viewpoint leads to classical Newton–Cotes and Gaussian quadrature rules. We show that many of these classical formulas also arise naturally from discretizations of certain boundary value problems. We exploit this connection to generate entirely new quadrature formulas and discuss ways to promote positivity.

Research Talk: A new framework for quadrature in geometrically complex domains
Tuesday 2:00-2:30

The dominant approach to constructing quadrature (or cubature) formulas is to select a "nice" vector space of functions for which the formulas are exact, such as algebraic or trigonometric polynomials. For one dimensional integration, this leads to classical Newton-Cotes and Gaussian quadrature rules. However, in higher dimensions and for geometrically complex domains, this exactness-based approach can be challenging or even infeasible, since it requires exact integration of basis functions over the domain or over suitable subdomains. Additional challenges arise when the integrand is known only through samples at predefined, possibly “scattered' points (i.e., a point cloud), as is common in applications involving experimental data or when integration is a secondary step in a larger computational process.

I will build upon my previous lecture to introduce a new framework for generating quadrature formulas that bypasses these challenges. The framework relies on numerical approximations of certain elliptic operators and on linear algebra. We demonstrate its ability to produce accurate quadrature formulas for geometrically complex domains, including point cloud discretizations of surfaces and unfitted discretizations of Euclidean domains.